Broadgate, London

A VaR framework for longevity trend risk

Stephen Richards
19th April 2012

Contents

1. Longevity trend risk...

- ...is the risk of an adverse long-term trend in improvements.
- Trend risk is only part of a collection of longevity-related risks.
- Ignoring misestimation and idiosyncratic risk for now.

2. General approach to capital requirement

- Capital requirement for longevity trend risk.
- Compare annuity factors using best-estimate and adverse mortality:

$$\left(\frac{\bar{a}_x^{\text{adverse}}}{\bar{a}_x^{\text{central}}} - 1\right) \times 100\%$$

- Use either specimen annuity factor or entire portfolio valuation.

2. General approach to capital requirement

- Problem: data only available to age 105 for UK population.
- Use temporary annuities:

$$\left(\frac{\bar{a}^{\text{adverse}}_{x:\overline{105-x}|}}{\bar{a}^{\text{central}}_{x:\overline{105-x}|}} - 1 \right) \times 100\%$$

Note that CBD and 2D P-spline models can extrapolate by age as well as projecting in time. See Richards and Currie (2011) for more details.

2. General approach to capital requirement

Consider three options for investigating impact of adverse mortality:

1. Stressed trend.
2. Mortality shock.
3. One-year value-at-risk.

3. Stressed trend

+ Operates over lifetime of annuity.
+ Correct approach for nature of risk.

– Not the one-year view demanded by Solvency II!

3. Stressed trend

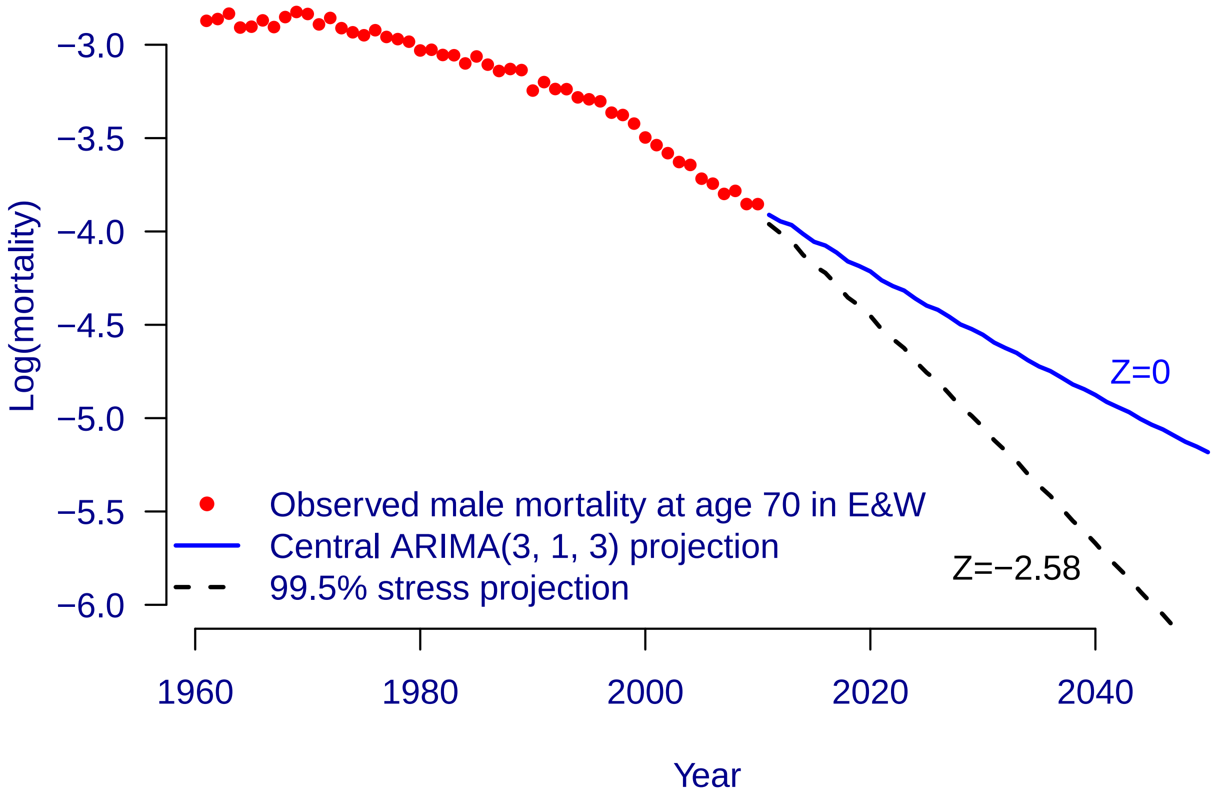
- Fit model to get best-estimate $\log \mu_{x,t}$
- ... and corresponding standard error s.e.($\log \mu_{x,t}$).
- Generate stressed trend from:

$$\log \mu_{x,t} - Z \times \text{s.e.}(\log \mu_{x,t})$$

- Z comes from the N(0,1) distribution, e.g. for 99.5% stress

$$\begin{aligned} Z &= \Phi^{-1}(0.995) \\ &= 2.58 \end{aligned}$$

3. Stressed trend



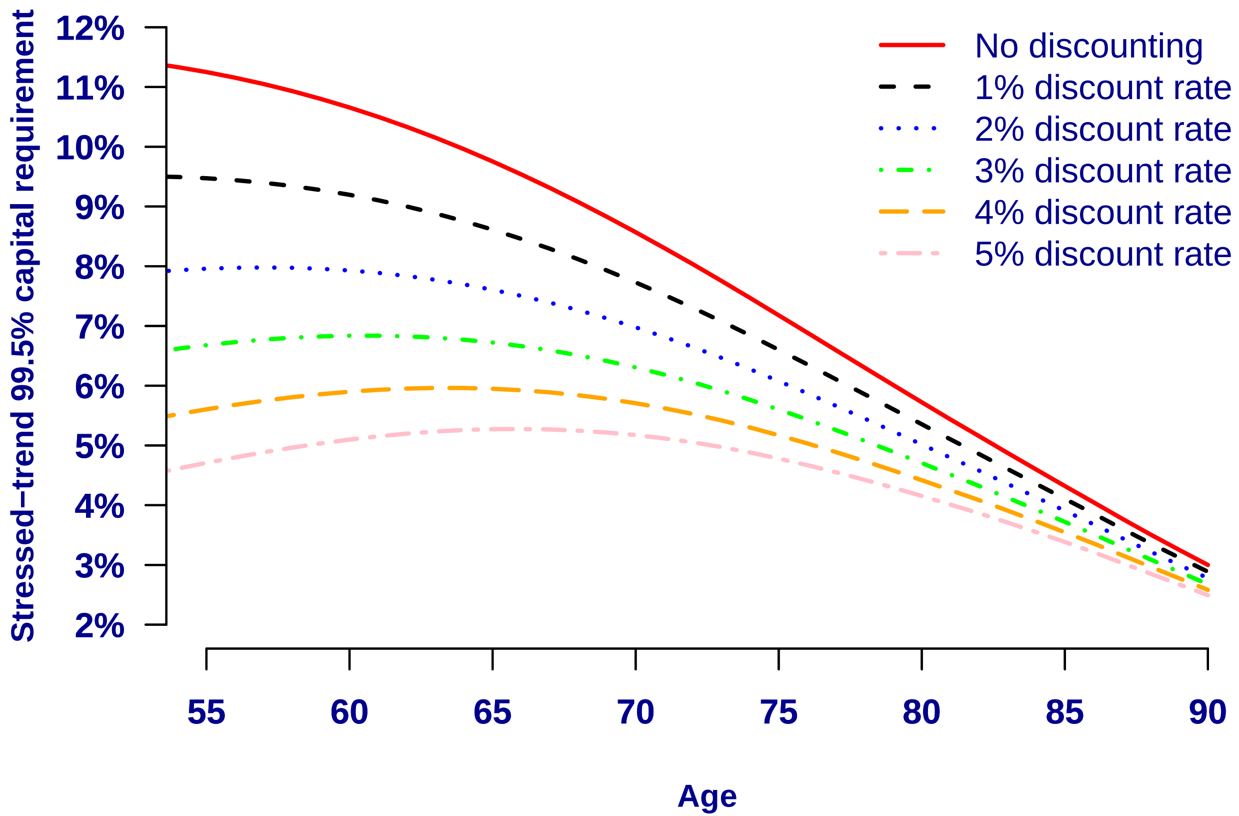
Source: Mortality of males aged 70 in England and Wales, modelled as per Lee and Carter (1992) with ARIMA(3,1,3) projection of κ_y.

3. Stressed-trend capital

Capital requirement from comparing annuity factors with and without stress:

$$\left(\frac{\bar{a}^{Z=-2.58}_{x:\overline{105-x}|}}{\bar{a}^{Z=0}_{x:\overline{105-x}|}} - 1\right) \times 100\%$$

3. Stressed-trend capital



Source: Mortality of males aged 70 in England and Wales, modelled as per Lee and Carter (1992) with ARIMA(3,1,3) projection of κ_y.

4. The role of discounting

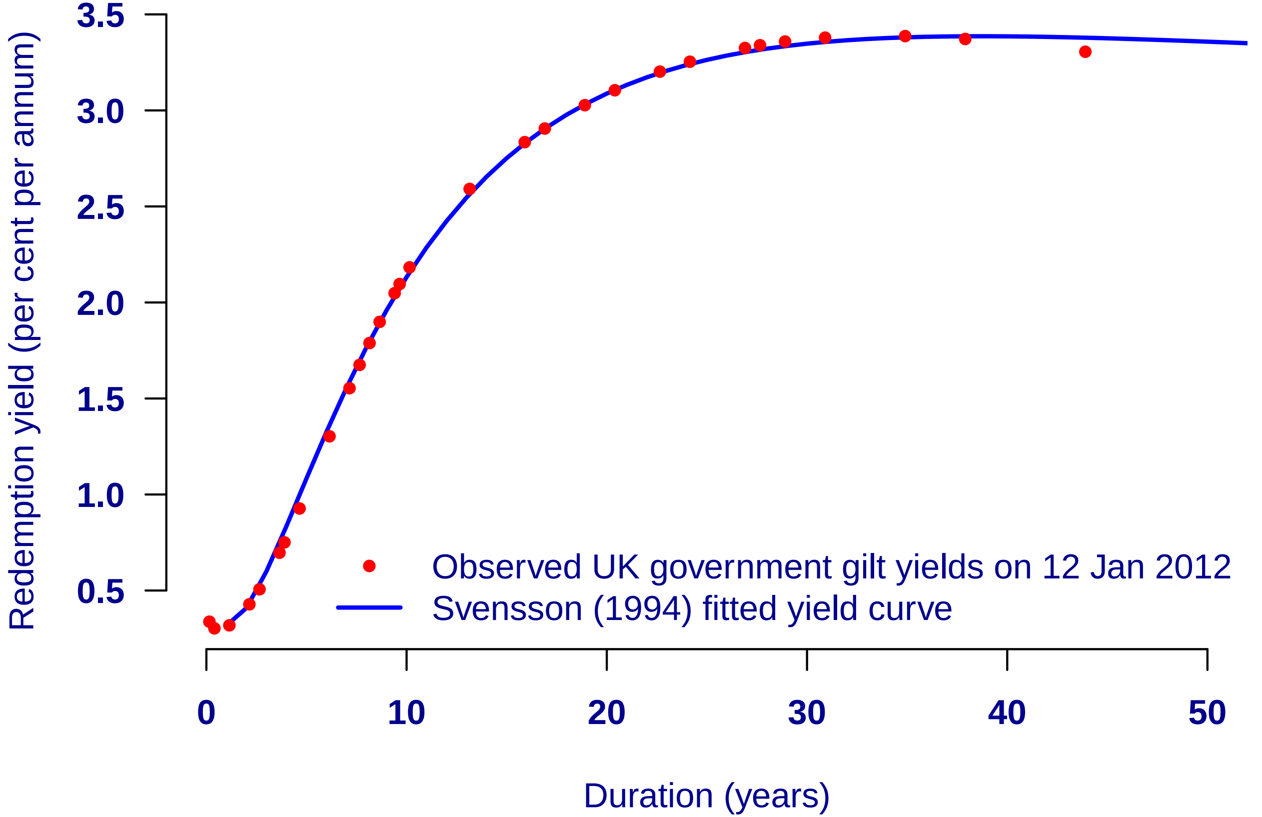
Annuity factor is defined as:

$$\bar{a}_{x:\overline{105-x}|} = \int_0^{105-x} {}_tp_x v^t dt$$

where ${}_tp_x$ is the survivor function and v^t the discount function.

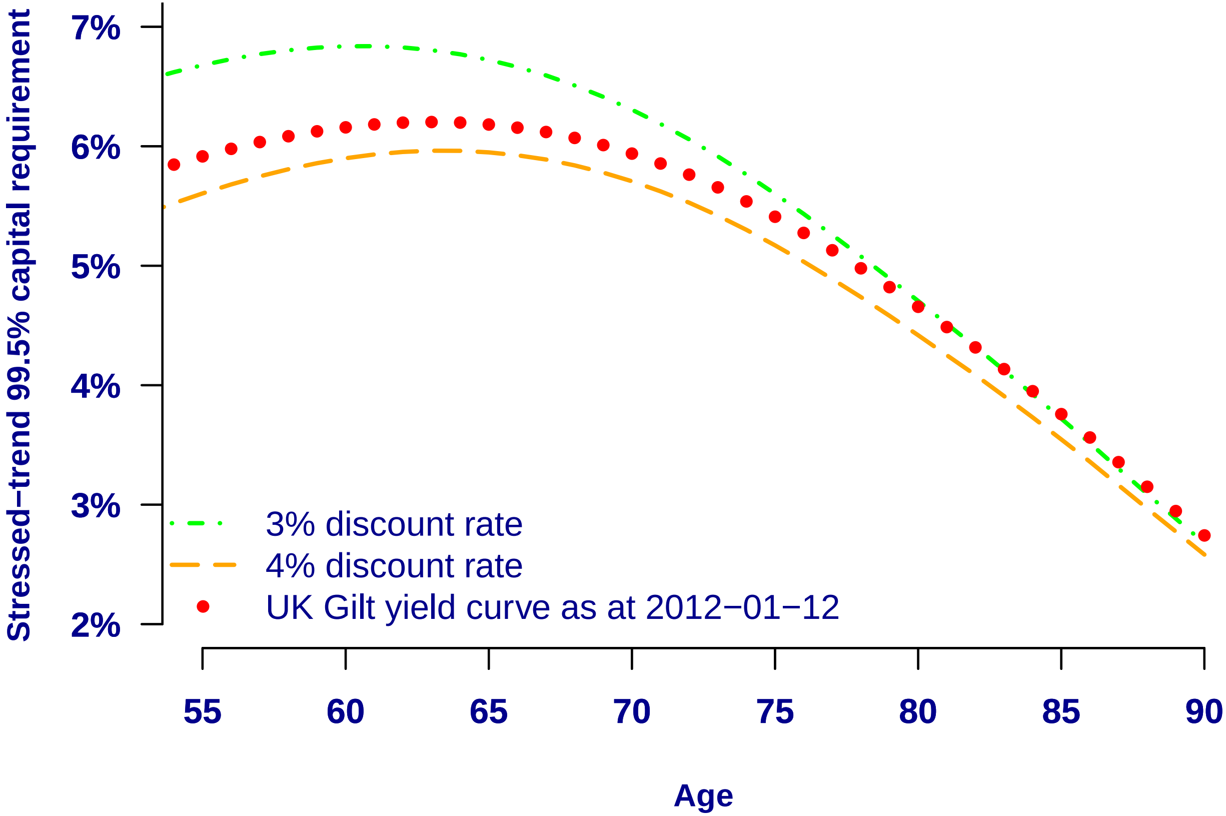
$\Rightarrow$ Longevity-trend capital requirement linked to discount function.

4. The role of discounting — yield curves



Source: Observed redemption yields implied by prices of principal strips of UK gilts on 12th January 2012 (●) and fitted Svensson (1994) model (——).

4. Stressed-trend capital — yield curves



Source: Own calculations using mortality of males aged 70 in England and Wales, modelled as per Lee and Carter (1992) with ARIMA(3,1,3) projection of κ_y with Svensson (1994) yield-curve model fitted to the yields implied by prices of principal strips of UK gilts on 12th January 2012.

5. Mortality shock

+ Operates over lifetime of annuity.
+ Very simple.
+ Corresponds to Solvency II standard formula.

– Produces unusual shape of capital requirements.

5. Mortality shock

Mortality shock, f:

$$\mu_{x,t}^{\text{shock}} = \mu_{x,t} \times (1 - f)$$

where $\mu_{x,t}$ is the central mortality projection and f is the shock reduction in mortality rates.

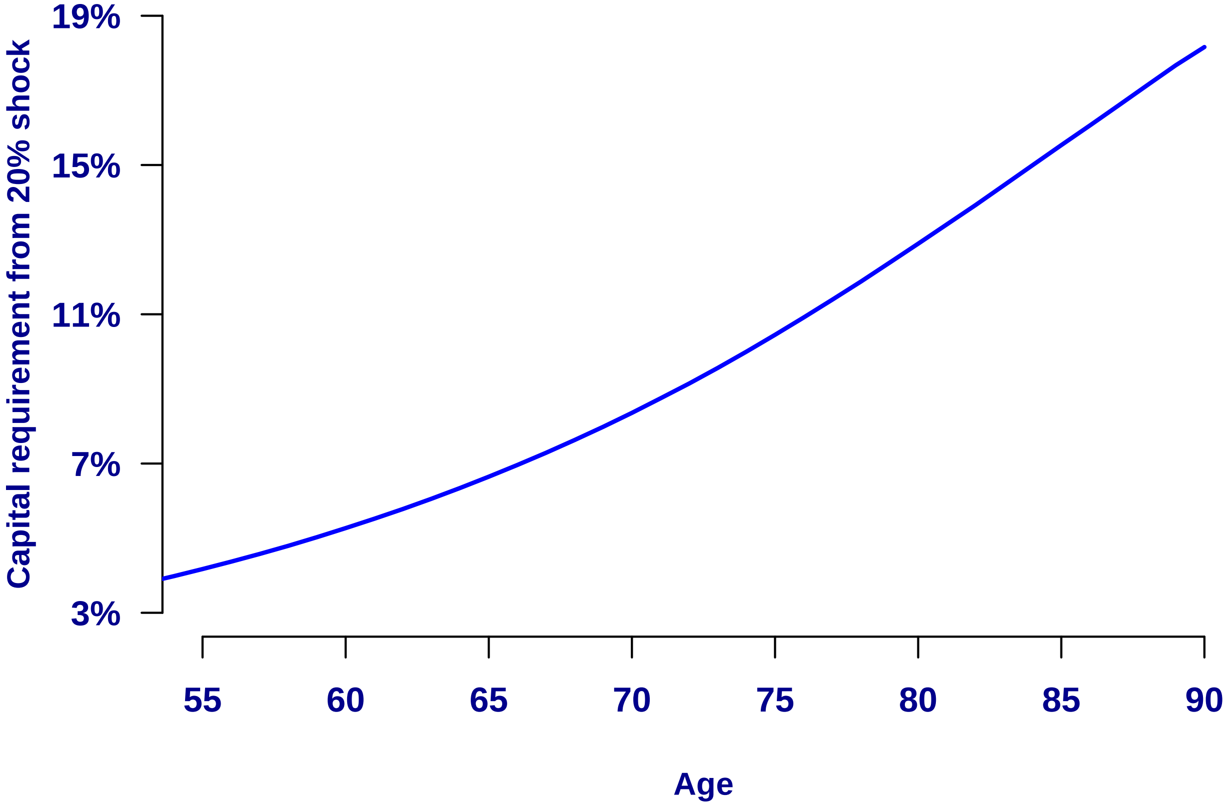
f was 25% in QIS4, but was relaxed to 20%

5. Mortality-shock capital

Capital requirement from comparing annuity factors with and without shock:

$$\left(\frac{\bar{a}^f_{x:\overline{105-x}|}}{\bar{a}_{x:\overline{105-x}|}} - 1\right) \times 100\%$$

5. Mortality-shock capital



Source: Central Lee-Carter projection from previous slides. Discounting at 3% per annum and a mortality shock of an immediate fall in mortality rates of 20%.

6. One-year value-at-risk

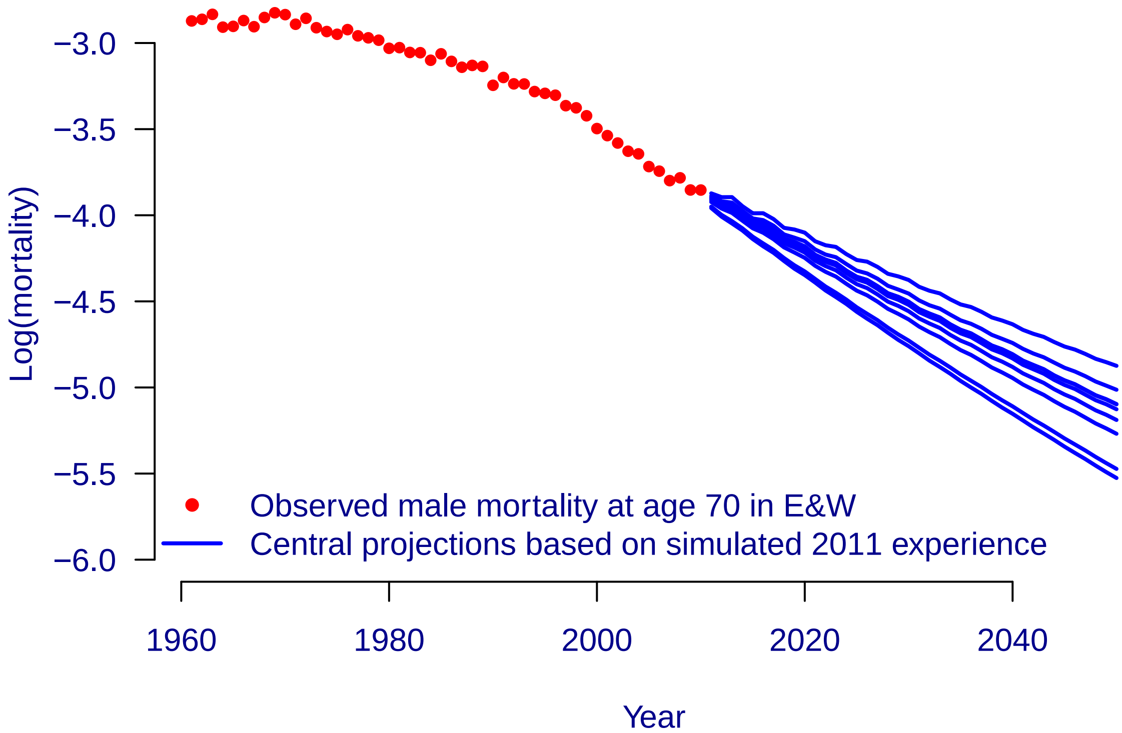
+ Operates over one year.
+ Corresponds to Solvency II value-at-risk view.

– More complicated to implement.

6. Recipe for one-year value-at-risk

1. Select a data set with end year Y
2. Select a statistical model and fit it to the data set in (1)
3. Use the statistical model in (2) to generate sample paths for $\log \mu_{x,Y+1}$
4. Simulate the number of deaths in year $Y+1$ at each age
5. Temporarily append simulated data from (4) to the real data in (1)
6. Refit model using pseudo-data and calculate annuity factor based on new central projection
7. Repeat (4)-(6) n times, where n should be at least 1,000 (say)

6. One-year value-at-risk



Source: Experience data for 2011 are simulated using sample paths from an ARIMA(3, 1, 3) process and the Lee-Carter model is refitted each time the 2011 data are simulated. The changes in central projections give an idea of how the best estimate could change over the course of a year based on new data.

6. One-year value-at-risk

From repeatedly updated projections we calculate a set of annuity factors, S:

$$S = \{\bar{a}_x^j ; j = 1, 2, \ldots, n\}$$

Value-at-risk capital requirement is then:

$$\left(\frac{99.5^{\text{th}} \text{ percentile of } S}{\text{mean of } S} - 1 \right) \times 100\%$$

6. One-year value-at-risk

Model	Value of $\bar{a}^{3\%}_{70:\overline{35}\rceil}$ (a) average value	(b) 99.5$^{\text{th}}$ percentile	Capital requirement $\left(\frac{(b)}{(a)} - 1\right) \times 100\%$
LC	12.22	12.83	4.99%
DDE	12.23	12.81	4.77%
LC(smooth)	12.22	12.81	4.83%
CBD	11.96	12.43	3.91%
APC	12.60	12.97	2.96%
2DAP	12.81	13.72	7.09%

Source: Richards, Currie and Ritchie (2012). Models: LC = Lee-Carter (1992), DDE = Delwarde, Denuit and Eilers (2007), LC(smooth) = Lee-Carter with smoothed α and β (Currie, 2012), CBD = Cairns, Blake and Dowd (2006), APC = Age-Period-Cohort and 2DAP = 2D penalised splines by age and period (Currie, Durban and Eilers, 2004).

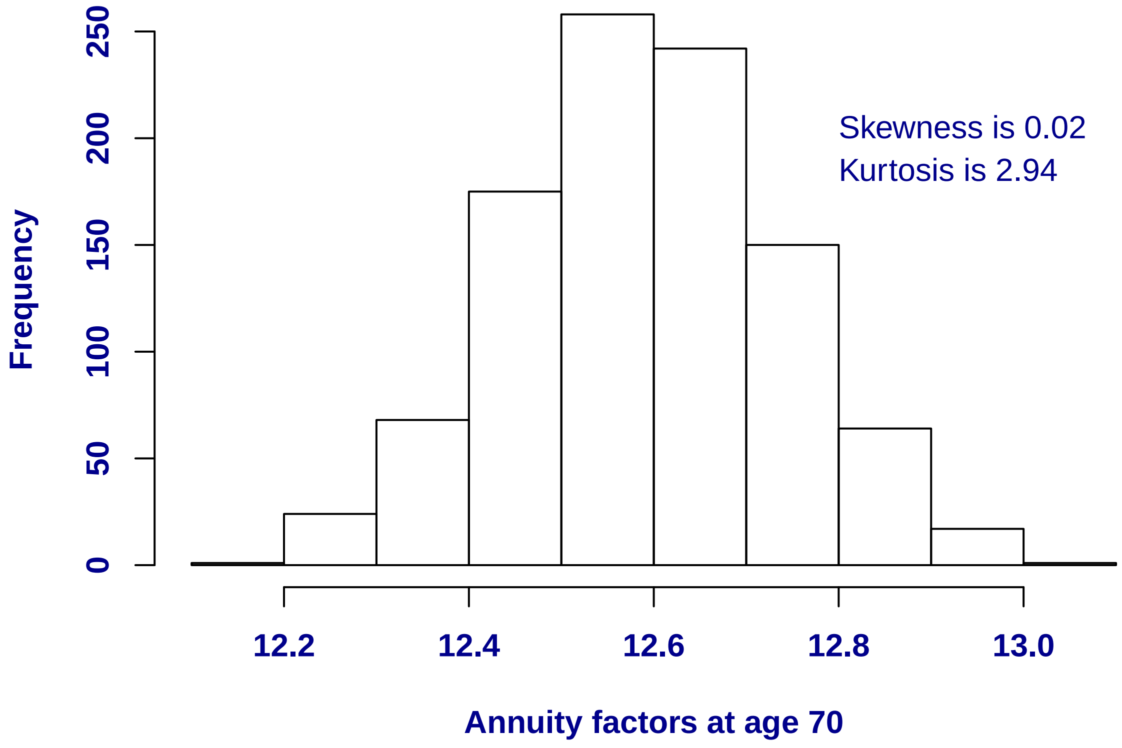
7. How many simulations?

- Ideally want more than 1,000 simulations...
- ...but VaR procedure time-consuming
- Several hours for 1,000 simulations!

7. How many simulations?

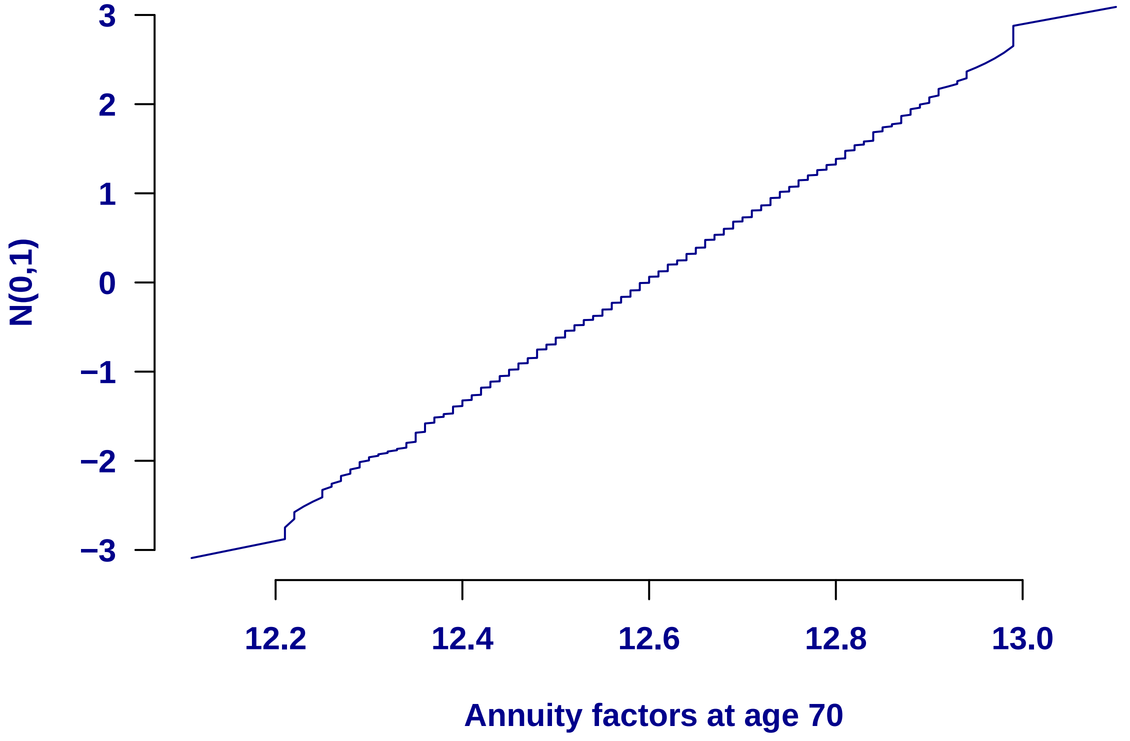
- Consider APC results.
- Capital from sample quantile is 2.96%
- Consider the distribution of annuity factors at age 70...

7. Distribution of annuity factors in one-year VaR



Source: Distribution of 1,000 annuity factors at age 70 based on updated APC model.

7. Q-Q plot of annuity factors in one-year VaR



Source: N(0,1) quantiles v. quantiles of 1,000 annuity factors at age 70 based on updated APC model.

7. How many simulations?

- Skewness is 0.02, kurtosis is 2.94 and Q-Q plot is linear

$\Rightarrow$ Plausible Normal distribution!

- Better capital estimate is then:

$$\begin{aligned} &\Phi^{-1}(0.995) \times \text{coefficient of variation} \\ &= 2.58 \times 1.16\% \\ &= 2.99\% \end{aligned}$$

7. How many simulations?

- Trick won't work in every situation.
- Consider Lee-Carter with smooth α and β at age 70.

$\Rightarrow$ assumption of normality yields capital of 4.30%.

- However, skewness is 0.63, kurtosis is 2.95.

$\Rightarrow$ Distribution not normal, so 4.30% figure is invalid.

- Sample quantile yields 4.83%.
- Sometimes you just have to do the simulations!

8. Model risk

- Nobody can say which model to use...
- ...although some have more question marks over them than others!
- Relying on a single model is unsafe.

9. Conclusions

- Stressed trend fits nature of risk, but isn't a one-year view.
- Standard formula produces unusual capital requirement by age.
- VaR approach fits one-year view…
- …with slightly lower capital requirements than stressed trend.
- Don't forget model risk — *must* use different models.

References — I

Cairns, A. J. G. and Blake, D. and Dowd, K. **2006** *A two-factor model for stochastic mortality with parameter uncertainty: theory and calibration*, Journal of Risk and Insurance, **73**, 687–718

Currie, I. D., Durban, M. and Eilers, P. H. C. **2004** *Smoothing and forecasting mortality rates*, Statistical Modelling, **4**, 279–298

Delwarde, A. and Denuit, M. and Eilers, P. H. C. **2007** *Smoothing the Lee-Carter and Poisson log-bilinear models for mortality forecasting: a penalized likelihood approach*, Statistical Modelling, **7**, 29–48

References — II

LEE, R. D. AND CARTER, L. **1992** *Modelling and forecasting the time series of US mortality*, Journal of the American Statistical Association, **87**, 659–671

LONGEVITAS LTD **2011** *Value at risk (VaR)*, Projections Toolkit Technical Guide v2.2, Chapter 16

RICHARDS, S. J. AND CURRIE, I. D. **2011** *Extrapolating mortality projections by age*, Life and Pensions Risk magazine, June 2011, 34–38

RICHARDS, S. J., CURRIE, I. D. AND RITCHIE, G. P. **2012** *A value-at-risk framework for longevity trend risk*, Longevitas Ltd